Optimal Capacitor Placement to Distribution Transformers for Power Loss Reduction in Radial Distribution Systems

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***Abstract*—Deploying shunt capacitor banks in distribution systems can effectively reduce the power loss and provide additional benefits for system operation. In practice, the power loss on distribution transformers can account for a considerable portion of the overall loss. This paper proposes a method for optimal placement of capacitor banks to the distribution transformers to reduce power loss. The capacitor bank locations are considered at the low-side of transformers. The net present value (NPV) criterion is adopted to evaluate the cost benefit of the capacitor installation project. First, an explicit formula for directly calculating the power loss of radial distribution systems is derived. Then, the optimal capacitor bank placement is formulated as a mixed-integer programming (MIP) model maximizing the NPV of the project subject to certain constraints. The model is suitable for being solved by commercial MIP packages, and the operational control of the capacitor banks to maximize the power loss reduction can be simply achieved by local automatic switching according to VAR measurements. The proposed method has been practically applied in the Macau distribution system, and the simulation results show that the proposed method is computationally efficient, and a considerable positive NPV can be obtained from the optimal capacitor bank placement.**

***Index Terms*—Capacitor placement, distribution transformer (TR), mixed-integer programming (MIP), net present value (NPV), radial distribution system.**

NOMENCLATURE

b_t	Annual saving due to capacitor installation ($).
B_t	Net annual profit ($).
C_t	Annual loss cost before capacitor installation ($).
$C_t^{'}$	Annual loss cost after capacitor installation ($).
d	Discount rate (%/year).
F_{loss}	Power loss factor.
I	Current magnitude that circulate through the line (A).
I_R	Real component of current I (A).
I_X	Reactive component of current I (A).
IO	Initial investment outlay of cash ($).
IC	capacitor installation cost ($).
K_E	Energy cost ($/kWh).
K_I	Capacitor installation cost for each TR ($)
K_P	Purchase cost of capacitor per unit size ($)
K_O	Annual O&M cost of capacitor banks for each TR ($).
l	Load growth rate.
L_i	Number of the capacitor modules in a capacitor bank (integer decision variable).
N	Total number of TRs in the network.
O&M	Operation and maintenance.
OM_t	Annual operation and maintenance cost of year t ($).
PC	Capacitor purchase cost ($).
p	Energy cost growth rate.
P_{Li}	Real power demand at TR i (kW).
P_{loss}	Power loss (kW).
$P_{\text{loss}}^{\text{TR}}$	Power loss at TR (kW).
$P_{\text{loss}}^{\text{Line}}$	Power loss at line section (kW).
$P_{\text{loss},R}$	Power loss caused by real current I_R (kW).
$P_{\text{loss},X}$	Power loss caused by reactive current I_X (kW).
$\bar{P}_{\text{loss},X}$	Average power loss (kW).
$\hat{P}_{\text{loss},X}$	Power loss at peak load (kW).
Q_C	Capacitor capacity of per unit size (kvar).
Q_i	Total reactive powers flowing out of node i (kvar).
Q_{Li}	Reactive power demand at TR i (kvar).

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R_i	Resistance of TR i (Ω).
$R_{i,i+1}$	Resistance of line section between node i and $i+1$ (Ω).
t	Year.
T	Project's expected life (year).
TR	Distribution transformer.
V_{1i}	Voltage magnitude of high side of TR i (V).
V_{2i}	Voltage magnitude of low side of TR i (V).
X_i	Binary decision variable (0, 1), indicating whether to install a capacitor bank at TR i.
ϕ	Power factor angle (degree).
\$	Dollar (per unit value).

I. INTRODUCTION

THE operation of a power distribution system is inevitably accompanied with power loss due to the *Joule* effect. This I^2R loss can be very large since it occurs throughout the conductors of the distribution system; as indicated in [1], it can account for 13% of the total power generation. Therefore, there have been strong incentives for utilities to try to reduce the power loss.

To reduce the I^2R loss in a distribution system, one approach is to shorten the overall network resistant path that the current is passing through. This can be achieved by altering the network topology, known as reconfiguration [2]. The second approach, on the other hand, is to reduce the branch current that comes from root buses to customers, as a common practice. This can be achieved by deploying shunt capacitor banks in the network to compensate a portion of reactive power demand of the loads [6]–[17]. In addition to saving power loss, capacitor installations can offer additional benefits, such as improving voltage profile, releasing network capacity, and providing reactive power reserve. This paper focuses on optimal capacitor placement in distribution networks for the interest of power loss reduction.

From a mathematical perspective, the optimal capacitor placement is a mixed-integer programming (MIP) problem, where capacitor location and size are to be optimized. In terms of objective function, most of the literature minimizes the total cost of capacitor installations minus the energy loss savings [6]–[13]; in the meantime, some proposals consider multiple objectives to account for voltage violations [14]. However, very few works appraise the investment project on a more realistic cost-benefit assessment basis. In terms of solution algorithms, heuristic search [6], Benders decomposition [7], [8], specifically tailored numerical programming techniques [9], genetic algorithms (GAs) [10], [11], [14], modified differential evolution (DE) algorithm [12], tabu search [13], and fuzzy expert systems [16] have been reported to solve the problem with success of varying degree. In addition, some works with more realistic problem modeling have also been reported, such as considering unbalanced system [9], harmonics [15], and uncertain and varying load [11]. A comprehensive review and discussion on previous works could be found in [14] and [17].

However, it is observed that while most of previous works consider only the power loss on line sections, very few systematically takes into account the loss on distribution transformers (TRs) in placing the capacitors. In fact, the power loss on the TRs makes up an appreciable portion of a utility's overall loss. According to [3], the TRs account for 26% of transmission and distribution losses and 41% of distribution and subtransmission losses [4]. In [5], it was estimated that the TRs occupy 55% of total distribution losses.

The power loss on a TR consists of load loss and no-load loss. Load loss corresponds to the I^2R loss, while no-load loss is caused by the eddy current and hysteresis occurring at the core material of the transformer. In this paper, only load loss is considered since no-load loss mainly depends on the manufacturing and materials. To reduce the load loss of a TR, one has to reduce the current passing through its windings. By installing capacitor banks at the low-side of the TR, a portion of the reactive power demand can be directly compensated, thereby reducing the current. In addition to reducing the TR loss, the decreased current passing through the TR windings can also reduce the overall power loss of the distribution network since it can diminish the branch current coming from the root bus. In this sense, the installation of capacitor banks to TRs can be viewed as a particular case of the general capacitor placement problem, but it can provide additional advantages, which are given here.

- It can effectively free up a large portion of capacity of the distribution system, especially the TRs. Hence, utilities can choose TRs of smaller size, incidentally decreasing the nonload loss (in general, the larger capacity of a TR, the larger nonload loss it can produce [3]).
- It can significantly simplify the operational control of the capacitor banks. For example, the control can be local automatic switching of taps according to the varying reactive power load, which is highly preferred from an engineering application perspective.
- It can provide local voltage boost to customer loads, which can cancel part of the drop caused by the varying loads.
- It will also be shown in this paper that the problem can be formulated as an explicit mixed-integer quadratic programming (MIQP) model without much assumption and approximation and suitable for being solved by commercial MIP packages.

In this paper, a method for optimal placement of capacitor bank to TRs in radial distribution networks for power loss reduction is proposed. To realistically appraise the cost benefit of the capacitor installation project, the net present value (NPV) criterion is applied. The objective is to maximize the NPV of the capacitor installation project considering the energy-saving benefits and various costs (capacitor purchase and installation cost and operating and maintenance cost) over the project lifecycle. Capacitor banks locations are considered at the low-side of the transformers to directly compensate the reactive power demand of the customer load. Based on an explicit formula to directly calculate the power loss of radial distribution systems, the optimal capacitor bank placement is formulated as a MIQP

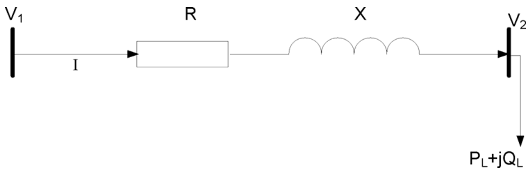

Fig. 1. Electrical equivalent model of a TR.

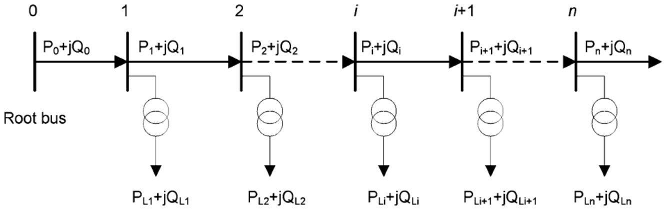

Fig. 2. Single-line diagram of a feeder in a radial distribution network.

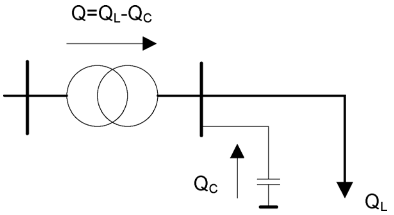

Fig. 3. Reactive power balance after capacitor installation to a TR.

model, which can be readily solved by high-performance commercial MIP packages such as CPLEX.[1] and GUROBI[2] The voltage constraint is satisfied through an iterative process. The proposed methodology has been applied in the Macau distribution system, and simulation results have demonstrated its effectiveness.

The remainder of this paper is organized as follows. Section II presents a formula for directly calculating the power loss in radial networks. Section III introduces the NPV criterion for appraising the cost benefit of the project. Section IV presents the mathematical model and its solution process. Section V presents a practical operational control strategy of the capacitor banks. Section VI presents the simulation results. Section VII concludes the whole paper.

II. DIRECT CALCULATION OF POWER LOSS IN RADIAL NETWORKS

A TR can be electrically modeled as Fig. 1, where I is the current passing through the TR, R and X are resistance and reactance of the TR, respectively, and V_1 and V_2 are voltage magnitudes at high- and low-side of the TR, respectively.

The power loss on a conductor can be decomposed into two parts, one caused by real current, and the other caused by reactive current, and shown as

$$\begin{aligned} P_{\text{loss}} &= \left(I_R^2 + I_X^2\right) \cdot R \\ &= I_R^2 R + I_X^2 R = P_{\text{loss},R} + P_{\text{loss},X}. \end{aligned} \tag{1}$$

For the TR shown in Fig. 1, the power loss is

$$\begin{aligned} P_{\text{loss}} &= P_{\text{loss},R}^{\text{TR}}(i) + P_{\text{loss},X}^{\text{TR}}(i) \\ &= \left(\frac{P_{Li}}{V_{i2}}\right)^2 \cdot R_i + \left(\frac{Q_{Li}}{V_{i2}}\right)^2 \cdot R_i. \end{aligned} \tag{2}$$

The capacitor bank, when installed at the low-side of the TR, can directly compensate the reactive power demand, thereby reducing $P_{\text{loss},X}$.

In this paper, the distribution network is assumed to be three-balanced, and current harmonics are not considered. Given a feeder with many TRs (see Fig. 2), the power loss $P_{X,\text{loss}}$ of line section connecting bus i and bus $i+1$ can be calculated by:

$$P_{X,\text{loss}}^{\text{Line}}(i, i+1) = \left(\frac{Q_i}{V_{i1}}\right)^2 \cdot R_{i,i+1} \tag{3}$$

where $R_{i,i+1}$ is the resistance of line section between node i and node $i+1$, V_{i1} is the voltage magnitude of node i and is equivalent to that of the high-side of TR i, and Q_i is the total reactive powers flowing out of node i and can be roughly accounted as

$$Q_i = \sum_{n=i+1}^{N} Q_{Ln} \tag{4}$$

where Q_{Ln} is the reactive power load at TR n.

The total power loss caused by reactive power demand of the system can then be explicitly calculated as

$$\begin{aligned} P_{\text{loss},X} &= \sum_{i=1}^{N} P_{\text{loss},X}^{\text{TR}}(i) + \sum_{i=0}^{N-1} P_{\text{loss},X}^{\text{Line}}(i, i+1) \\ &= \sum_{i=1}^{N} \left(\frac{Q_{Li}}{V_{i2}}\right)^2 \cdot R_i + \sum_{i=0}^{N-1} \left(\frac{\sum_{n=i+1}^{N} Q_{Ln}}{V_{i1}}\right)^2 \cdot R_{i,i+1} \end{aligned} \tag{5}$$

where V_1 and V_2 are determined by running initial power flow.

The annual cost ($) due to the power loss is calculated by

$$C_t = \hat{P}_{\text{loss},X} \cdot F_{\text{loss}} \cdot K_E \cdot 8760 \tag{6}$$

where K_E is the energy cost ($/kWh) and F_{loss} is the power loss factor which is the ratio between the average power loss and the peak power loss and is given as

$$F_{\text{loss}} = \frac{\bar{P}_{\text{loss},X}}{\hat{P}_{\text{loss},X}}. \tag{7}$$

To compute F_{loss}, a segment of historical load profile over a certain period (e.g., last one year) is obtained from the metering database, and the power loss at each time point is calculated by running power flow. The peak power loss $\hat{P}_{\text{loss},X}$ is the power loss at the peak load point and the average power loss $\bar{P}_{\text{loss},X}$ is the average value of all of the time points.

As Fig. 3 illustrates, when installing capacitor bank at the low-side of the TR, the reactive power passing through the TR windings can be reduced by the capacitor capacity.

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[1] [Online]. Available: http://www-01.ibm.com/software/integration/optimization/cplex-optimizer/

[2] [Online]. Available: http://www.gurobi.com/

The power loss after capacitor bank installation becomes

$$P'_{\text{loss},X} = \sum_{i=1}^{N} \left(\frac{Q_{Li} - L_i \cdot Q_C \cdot X_i}{V_{i2}} \right)^2 \cdot R_i + \sum_{i=0}^{N-1} \left(\frac{\sum_{n=i+1}^{N} (Q_{Ln} - L_n \cdot Q_C \cdot X_n)}{V_{i1}} \right)^2 \cdot R_{i,i+1} \quad (8)$$

where Q_C is the capacity of capacitor per unit size, L_i is the integer variable representing the number of the capacitor modules in a bank, and X_i is the binary decision variable indicating whether to install the capacitor bank at TR i (1: yes; 0: no). The product of L_i, Q_C, and X_i equals the reactive power compensation capacity to the TR.

III. NPV ANALYSIS

To practically evaluate the economic value of the capacitor installation project, one needs to compare the expected revenue and investment costs over the whole project lifecycle.

In this paper, the NPV criterion is adopted for cost-benefit analysis of the project. The NPV discounts each year's cash flow back to the present and then deducts the initial investment, giving a net value of the project in today's dollars. When the NPV is positive, the project can be accepted since it means the project can add value to the utility; otherwise, the project should be rejected because it will subtract the value to the utility. The NPV criterion can appraise a long-term project with the following advantages [18].

- It deals with cash flows rather than accounting profits.
- The accepted project will increase the value of the utility, since only the projects with positive NPV are accepted.
- It recognizes the time value of money and allows for comparison of the benefits and costs in a logical manner.
- It can incorporate risk into the assessment of a project, either by adjusting the expected cash flows or by adjusting the discount rate.

After the capacitor banks installation, the new annual cost in dollars is

$$C'_t = \hat{P}'_{\text{loss},X} \cdot F_{\text{loss}} \cdot K_E \times 8760. \quad (9)$$

The annual savings by applying capacitor bank to TRs is

$$b_t = C_t - C'_t. \quad (10)$$

It is worth mentioning that other savings such as those produced by released network capacity can also be added in (10), but this paper only considers the power loss reduction.

Considering the O&M costs OM_t of the capacitor banks, the net annual profit should be

$$B_t = b_t - \text{OM}_t. \quad (11)$$

OM_t is calculated by

$$\text{OM}_t = \sum_{i=1}^{N} X_i \cdot K_O \quad (12)$$

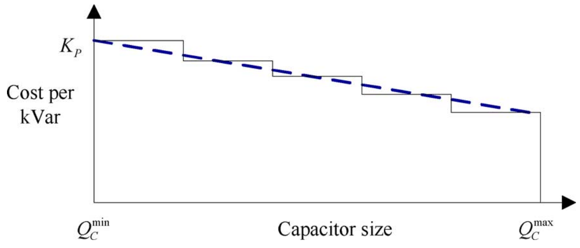

Fig. 4. Capacitor cost function.

where K_O is the annual O&M cost of capacitor banks for each TR in dollars.

For the capacitor installation project, the NPV can be calculated as follows:

$$\text{NPV} = \sum_{t=1}^{T} \frac{B_t}{(1+d)^t} - IO \quad (13)$$

where B_t is the annual net cash flow in year t, d is the discount rate, T is the project's expected life, and IO is the total initial investment outlay of cash including capacitor purchase cost and installation cost and is given as

$$IO = IC + PC \quad (14)$$

where

$$IC = \sum_{i=1}^{N} X_i \cdot K_I \quad (15)$$

$$PC = \sum_{i=1}^{N} L_i \cdot X_i \cdot K_P \quad (16)$$

where K_P and K_I stand for the purchase cost of the capacitors per unit size and installation cost for each TR, respectively.

Equation (16) is applied when only a fixed capacitor bank unit size is adopted. In practice, capacitor banks of larger unit size can have lower per kVar price. Hence, capacitor banks with different unit sizes may be combined during installation.

Generally, the capacitor cost function is a piecewise function as the real line shows in Fig. 4.

To consider the varying purchase cost of the capacitor depending on the capacitor unit size, the piecewise cost function can be approximated by a linear function as the dash line shows in Fig. 4.PC can then be calculated by

$$PC = \sum_{i=1}^{N} L_i \cdot X_i \cdot (K_P - \gamma L_i) \quad (17)$$

where γ is the equivalent slope of the linearized cost function, and K_P here corresponds to the minimum capacitor unit size $Q_C^{\min}$.

IV. MODEL FORMULATION AND SOLUTION

The mathematical model of the capacitor installation project is to maximize the NPV subject to certain constraints so as to obtain the maximum economic benefits in terms of investment and revenue. The mathematical model is presented as follows:

$$\max \quad \text{NPV} = \sum_{t=1}^{T} \frac{(C_t - C'_t) - OM_t}{(1+d)^t} - (PC + IC) \tag{18}$$

subject to the following.

- For fixed capacitor:

$$0 \le L_i \cdot Q_C \cdot X_i \le Q_{Li}^{\min}. \tag{19}$$

- For controlled capacitor:

$$Q_{Li}^{\min} \le L_i \cdot Q_C \cdot X_i \le Q_{Li}^{\max}. \tag{20}$$

The NPV is the sum of the cash flow of each year in the expected project lifetime; thus, the growth of load demand and energy cost should also be considered in calculating the term for each year. The constraint (19) means that, if the capacitor size is fixed, the capacity should be less than the minimum reactive power demand at the TR, and (20) means that, if the capacitor size is adjustable, e.g., automatically switched by the controller, the capacity should be between the minimum and maximum reactive power load of the TR.

It can be seen that due to the multiplying of the decision variables L and X in (8), (16), (17), (19), and (20), the problem constitutes a mixed-integer nonlinear programming (MINLP) model which is difficult to solve. We then rewrite the model to be a MIQP formulation, which is much easier to solve. To this end, the binary decision variable X in (8), and (16), and (17) is removed and constraints (19) and (20) are rewritten as

$$0 \le L_i \cdot Q_C \le Q_{Li}^{\min} \cdot X_i \tag{21}$$
$$Q_{Li}^{\min} \cdot X_i \le L_i \cdot Q_C \le Q_{Li}^{\max} \cdot X_i. \tag{22}$$

In such a way, the model becomes a MIQP one while the same mathematical characteristic is maintained.

It is worth mentioning that, although the quadratic terms can be further linearized to yield a mixed-integer linear programming (MILP) model that is further easier to solve, we retain the quadratic formulations since high-performance MIQP solvers are currently available in most commercial packages such as CPLEX and GUROBI .

It should be noted that the voltage constraint is not directly included in the optimization model. Rather, the voltage is satisfied through an iterative process shown in Fig. 5.

Before capacitor deployments, the bus voltage should be already regulated at a normal level. After capacitor banks are installed, the voltage magnitude of the low-side of TR is generally improved rather than degraded since a portion of reactive power load is compensated. Hence, it is usually needed to examine only the overvoltage case after capacitor installed. Actually, the voltage boosting is rather limited since the size of capacitor bank is constrained by (19) and (20). According to Fig. 5, once the optimization results are obtained, power flow simulations are then performed to examine if overvoltage appears (note that both peak and bottom load conditions can be examined, for automatically switched capacitor banks, their output in bottom load condition should be accordingly decreased to avoid inverse reactive power injection). If the overvoltage occurs for some load buses, the voltage can be regulated to normal level by adjusting the local customer TR tap changers—this can be implemented during capacitor installation stage. Generally, a TR tap changer has a broad range to adjust, however, when the TR tap changer reaches limits and can no longer regulate the voltage, it is then needed to modify constraints (19) and (20) by a smaller upper limit and resolve the optimization model. This process iterates until all of the voltage constraints are satisfied.

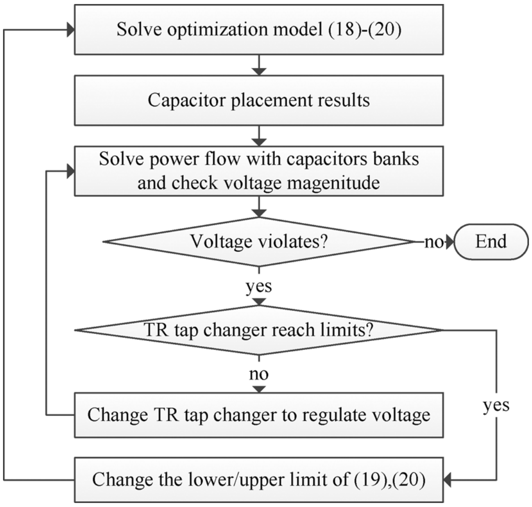

Fig. 5. Flowchart for satisfying voltage constraints.

V. OPERATIONAL CONTROL OF CAPACITOR BANKS

As already mentioned, the installation of capacitor banks to TRs can simplify the operational control of the capacitor banks. Unlike complicated coordination of switching actions of the capacitors in a distribution system [9], the control of the TR capacitor banks to maximize the reduction of the power loss can be approximated by locally switching the capacitor series according to the reactive power load sensed by the capacitor controller [3]. A simple switching strategy can be as follows.

Operational control rule

- For $Q_{Ci} < Q_{Li}$:
  Switch up to a tap that minimizes $|Q_{Ci} - Q_{Li}|$.
- For $Q_{Ci} > Q_{Li}$:
  Switch down to a tap that minimizes $|Q_{Li} - Q_{Ci}|$.

Here, Q_{Li} is the reactive power demand at TR I, sensed by the controller, and Q_{Ci} is the output of the capacitor bank.

The above control strategy can locally minimize the reactive current of each TR and, thus, can almost minimize the reactive current of line sections which is the sum of the reactive currents of the individual TRs. This control strategy can also contribute to voltage regulation as it avoids inverse reactive power injection during light load conditions.

VI. SIMULATION RESULTS

The proposed methodology has been practically implemented for power loss reduction of Macau medium voltage (MV) distribution network. MV herein means the voltage level from the

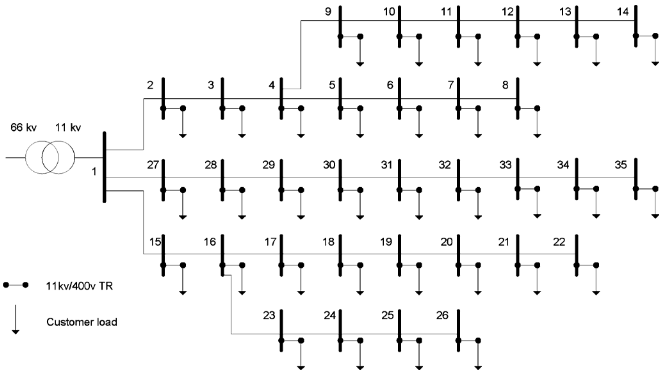

Fig. 6. One-line diagram of the test system.

TABLE I
PARAMETERS OF LINE SECTIONS OF THE STUDIED NETWORK

Line section		R (p.u.)	X (p.u.)	B (p.u.)
Sending end	Receiving end			
1	2	0.009686	0.015187	0.000184
2	3	0.006954	0.010904	0.000132
3	4	0.006209	0.009736	0.000118
4	5	0.011672	0.018303	0.000222
5	6	0.011672	0.018303	0.000222
6	7	0.003229	5.06E-03	6.15E-05
7	8	0.007699	0.012072	0.000147
4	9	0.003229	5.06E-03	6.15E-05
9	10	0.017136	0.02687	0.000326
10	11	0.009686	0.015187	0.000184
11	12	0.006954	0.010904	0.000132
12	13	0.006209	0.009736	0.000118
13	14	0.011672	0.018303	0.000222
1	15	0.040977	0.068543	0.000822
15	16	0.029802	0.04985	0.000598
16	17	0.010927	0.018278	0.000219
17	18	0.023593	0.039464	0.000473
18	19	0.007947	0.013293	0.000159
19	20	0.0226	0.037803	0.000453
20	21	0.00596	0.00997	0.00012
21	22	0.000993	0.001558	1.89E-05
16	23	0.004222	0.00662	8.04E-05
23	24	0.006209	0.009736	0.000118
24	25	0.006954	0.010904	0.000132
25	26	0.00447	0.00701	8.51E-05
1	27	0.009934	0.015577	0.000189
27	28	0.00298	0.004673	5.67E-05
28	29	0.009934	0.015577	0.000189
29	30	0.005464	0.008567	0.000104
30	31	0.011424	0.017913	0.000217
31	32	0.001987	0.003115	3.78E-05
32	33	0.006457	0.010125	0.000123
33	34	0.00298	0.004673	5.67E-05
34	35	0.00298	0.004673	5.67E-05

11 kV side of a 66/11 kV transformer to 400 V side at a 11-kV distribution transformer of the Macau distribution system.

The simulation is conducted on a 32-bit PC with 2.53-GHz CPU and 2G RAM. The commercial MIP package GUROBI is used to solve the optimization model.

To illustrate the effectiveness of the proposed method, its application to a portion of the network with five feeders/laterals and 34 TRs is presented here. This network can be viewed as a 69-bus system; its one-line diagram is shown in Fig. 6, and its parameters are given in Tables I and II. The base MW is 100 MW, the base voltage is 11 kV, and the voltage set-point of bus #1 is 1.01 p.u. Other relevant parameters involved in the optimization are given in Table III. Stated otherwise, the values in Table III do not necessarily reflect the reality of the Macau distribution system.

First, the proposed formula for direct calculation of power loss of radial distribution systems is verified. The initial overall

TABLE II
PARAMETERS OF TRS OF THE STUDIED NETWORK

TR Bus	Peak load		Bottom load		R (p.u.)	X (p.u.)
	P (kW)	Q (kVar)	P (kW)	Q (kVar)		
2	19	11.7	8.0	2.6	0.71875	0.06
3	409.1	254.2	50.8	37.2	1.28	0.06
4	260.9	203	82.2	56.2	0.71875	0.06
5	547.7	243.3	37.7	30.2	0.71875	0.06
6	185.4	153.4	14.2	12.6	0.71875	0.06
7	260.1	146.1	19.7	15.6	1.839254	0.04
8	445.1	157.4	183.7	99.5	1.28	0.06
9	493.1	203	190.2	83.7	1.28	0.06
10	186.3	67.7	68.0	26.9	1.28	0.06
11	342.8	149.3	114.8	55.4	1.839254	0.04
12	319.8	113.1	93.2	24.2	1.839254	0.04
13	68.2	21.9	31.5	9.9	1.28	0.06
14	244.2	87	116.3	42.3	1.28	0.06
15	499.4	196.5	445.8	168.9	0.71875	0.06
16	717.5	406.1	439.6	212.2	0.71875	0.06
17	659.5	431.7	389.8	345.0	0.71875	0.06
18	765.6	348.4	304.7	158.7	0.71875	0.06
19	873.5	551.9	306.2	214.9	0.71875	0.06
20	830.6	253	456.2	154.1	0.71875	0.06
21	988	280.5	410.2	119.5	0.71875	0.06
22	346.6	163	101.3	64.7	1.28	0.06
23	138.8	45.9	62.9	20.3	1.839254	0.04
24	183	41.2	55.7	11.1	1.839254	0.04
25	839.6	190.7	267.9	64.2	0.71875	0.06
26	13.9	4.3	5.1	1.2	0.71875	0.06
27	112.5	14	62.6	5.6	1.28	0.06
28	86.1	16.2	8.5	3.0	1.28	0.06
29	774.6	358.1	81.5	44.9	0.71875	0.06
30	461.1	175.8	53.1	11.2	0.71875	0.06
31	242.2	59.5	125.7	40.6	0.71875	0.06
32	48.1	0.1	66.3	3.0	1.839254	0.04
33	348.2	106.7	140.3	52.4	1.28	0.06
34	0.7	0.8	0.5	0.6	1.28	0.06
35	394.8	126.6	111.7	65.5	1.28	0.06
Total	**13106**	**5582.1**	**4906.0**	**2257.8**		

TABLE III
PARAMETERS IN CALCULATION

Parameter	Sign	Value
Minimum unit size of capacitor (kVar)	$Q_C^{\min}$	25
Purchase cost of capacitors per unit size ($Q_C^{\max}$) ($)	K_P	5000
Slope of the linearized capacitor purchase cost function	γ	30
Capacitor installation cost for each TR ($)	K_I	7500
O&M cost of capacitor for each TR ($/year)	OM_t	800
Energy cost ($/kWh)	K_E	1.136
Project lifetime (year)	T	10
Loss factor	F_{loss}	0.554
Inflation rate	p	5.0%
Discount rate	d	7.0%
Load growth rate	l	6.7%

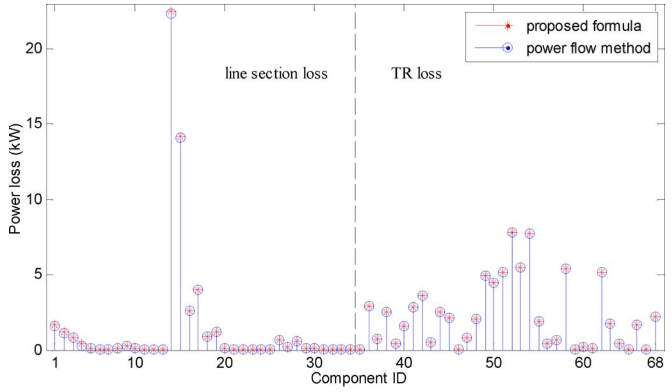

Fig. 7. Initial power loss at each component of the studied network.

TABLE IV
CAPACITOR PLACEMENT OPTIMIZATION RESULTS

TR bus	X	Capacity (kVar)
3	1	200
5	1	175
7	1	125
9	1	150
11	1	125
16	1	375
17	1	400
18	1	325
19	1	525
20	1	225
21	1	250
22	1	150
29	1	275
Others	0	0
Total	**13**	**3300**

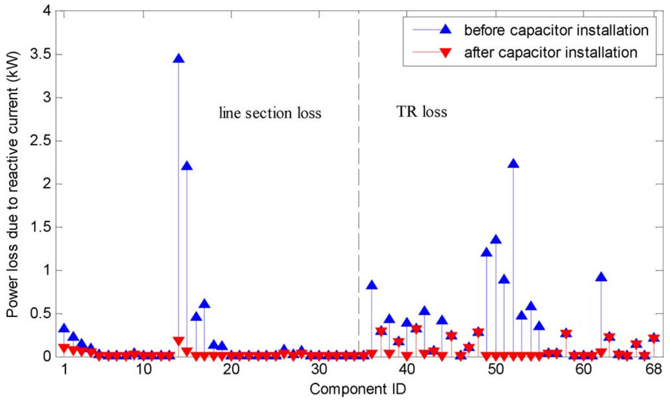

Fig. 8. Peak power loss ($\hat{P}_{\mathrm{loss},X}$) of each component before and after capacitor installations.

TABLE V
SUMMARY OF OPTIMIZATION RESULTS

	Before	**After**
$\hat{P}_{loss,X}$ (kW)	20.88	3.27
Power factor	0.913	0.951
Total initial invest ($)	708,660	
Total benefit ($)	1,896,865	
NPV ($)	1,188,205	
Computation time (s)	0.64	

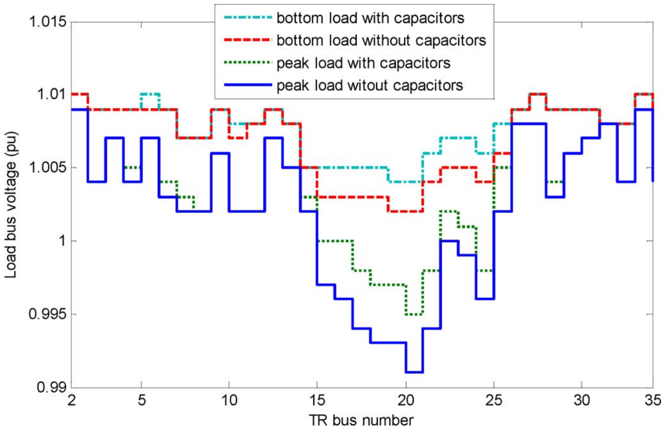

Fig. 9. Load bus voltage magnitudes in different conditions.

power loss of the studied network is respectively calculated by power flow method and the proposed formula, and the results are 129.9 and 130.5 kW, respectively, yielding a very small overall percentage error of 0.46%. The calculated loss of each component of the studied network is shown in Fig. 7, where ID 1–34 for line sections and ID 35–68 for TRs. It can be seen that the calculation error of the proposed formula only occurs at line sections, and the accuracy is sufficiently high for practical use.

It is also worth mentioning that the power loss on TRs accounts for a significant portion of the whole power loss in this feeder: 58.9%.

Using the proposed method, the optimal capacitor placement scheme for the studied network is calculated. The optimization results are given in Table IV. A total of 13 TRs are installed with capacitor banks and the total capacity is 3300 kVar.

Table V summarizes the optimization results. It can be seen that, after the capacitor installations, the peak power loss is significantly reduced, and the power factor is improved in the meantime. Fig. 8 shows the power loss caused by reactive current $\hat{P}_{\mathrm{loss},X}$ before and after the capacitor installations for each component. It can be seen that the capacitor banks have not only reduced the loss on TRs but also line sections. In the economic aspect, the initial investment (IO) of the project is $708,660, and the total benefit is $1,896,865, yielding a positive NPV of $1,188,205, which means that the project can add a net value of $1,188,205 to the utility over ten years.

The load bus voltage magnitudes are also examined, and the results are shown in Fig. 9. It can be seen that, after the capacitor deployment, the system voltage level is improved, especially for the buses where a capacitor is installed, and no overvoltage appears.

In addition, it is worth mentioning that the proposed method is quite computationally efficient, as the solution time of the model using GUROBI for the studied network is only 0.64 s.

VII. CONCLUSION

Power loss due to the *Joule* effect in a distribution system can be very large, where the loss on TRs can account for a considerable portion. This paper proposes a method for optimal placement of capacitor banks to TRs for power loss reduction in radial distribution systems. The problem is modeled as maximizing the NPV of the capacitor installation project subject to certain constraints and is formulated as an MIP model based on an explicit formula for direct calculation of the power loss of the radial distribution system. The model can be solved by commercial MIP packages very efficiently. The proposed methodology has been practically implemented in Macau MV distribution system. Its application to a portion of Macau system is illustrated in this paper, and the results show that by installing capacitor banks at optimized locations, the power loss of the network can be significantly reduced, the voltage level can be improved, and a positive large NPV can be obtained, which adds values to the utility.

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